Collapse-Induced Excitation Redistribution and Entropy Structuring in Fully Connected Qubit Networks

Nicholas Panek

Independent Researcher

Abstract

We study measurement-induced collapse in a 16-qubit system initialized in a uniform superposition over computational basis states with total Hamming weight at least 8. After an all-to-all controlled-Z (CZ) layer, we measure 8 qubits in Z and analyze the conditional state of the unmeasured 8 qubits. A strict tradeoff appears between expected total excitations and entropy. The outcome with zero observed excitations on the measured side deterministically prepares the unmeasured register in $|11111111\rangle$, giving Eq=1 for every unmeasured qubit, total E=8, and zero entropy. All reported Z-basis statistics follow from the global constraint and post-selection. Because CZ is diagonal in Z, it does not change these statistics. We outline how to expose phase effects by adding interference.

Introduction

Quantum measurement reshapes the state of an entangled system. When the global state space is restricted by a Hamming-weight threshold, post-selection drives strong redistribution of excitations. We analyze a 16-qubit register prepared uniformly over all bitstrings with total weight 8 or higher, apply all-to-all CZ, measure 8 qubits in Z, and study excitation and entropy on the remaining 8. We find a deterministic extreme at one end and a smooth energy-entropy trend across all outcomes.

System and Methods

Initial State

The initial state $|\psi 0\rangle$ is the uniform superposition on all 16-bit strings with Hamming weight ≥ 8:

$$|\psi 0\rangle=(1/N')\sum|x\rangle$$

where the sum is over all x with $|x|\geq 8$, and N' is the normalization factor:

$$N'=\sum k=816C(16,k)=39,203$$

Entangling Layer

Apply CZ on every pair of qubits (120 gates total). CZ is diagonal in the computational basis and applies a phase flip on $|11\rangle$. All CZ gates commute. Z-basis probabilities are unchanged.

Measurement

Measure qubits 0-7 in Z, obtaining a bitstring of Hamming weight r ($0 \le r \le 8$).

Metrics

For each unmeasured qubit (positions 8-15):

- $Eq(r)$: probability the qubit is in state $|1\rangle$
- Total excitations: $E = \sum Eq = 8 \times Eq(r)$ (by symmetry)
- Binary entropy: $hz(p) = -p \log_2 p - (1-p)\log_2(1-p)$
- Marginal entropy: $Hmarg = 8 \times hz(Eq)$
- Joint entropy: $Hjoint = \log_2 S(r)$, where $S(r)$ is the count of allowed unmeasured strings
- Multi-information: $I = Hmarg - Hjoint$

Core Combinatorics

With $L = 8$ unmeasured qubits and threshold $t = \max(0, 8-r)$:

$$S(r) = \sum_{k=t}^{8} C(8,k)$$

Expected Excitations per Qubit

$$Eq(r) = [\sum_{k=t}^{8} k \times C(8,k)] / [8 \times S(r)]$$

This can be equivalently written as:

$$Eq(r) = [\sum_{j=t-1}^{7} C(7,j)] / S(r)$$

Probability Distribution

$$P(r) = C(8,r) \times S(r) / 39{,}203$$

Results

State Counts and Entropies

The results are summarized in Figure 1 and the table below:

r	S(r)	Eq(r)	E	Hmarg	Hjoint	I	P(r)
0	1	1.000	8.000	0.000	0.000	0.000	2.55×10−5
1	9	0.889	7.111	4.026	3.170	0.856	0.00184
2	37	0.784	6.270	6.026	5.209	0.816	0.0264
3	93	0.688	5.505	7.162	6.539	0.623	0.133
4	163	0.607	4.859	7.732	7.349	0.383	0.291
5	219	0.548	4.384	7.947	7.775	0.172	0.313
6	247	0.514	4.113	7.995	7.948	0.047	0.176
7	255	0.502	4.016	8.000	7.994	0.006	0.0520
8	256	0.500	4.000	8.000	8.000	0.000	0.00653

Key Observations

- **Deterministic extreme (r=0):** When no excitations are observed in the measured qubits, the constraint forces t=8, giving S=1, Eq=1, E=8, Hmarg=0, Hjoint=0. The unmeasured register is deterministically in $|11111111\rangle$. This occurs with probability 2.55×10−5 (≈1 in 39,203 shots).
- **Uniform extreme (r=8):** When all measured qubits show excitations, there's no constraint on unmeasured qubits (t=0), giving S=256, Eq=0.5, E=4, Hmarg=8, Hjoint=8. This represents maximum entropy. Probability: 0.653%.
- **Monotone trends:** As r increases from 0 to 8:
  - Eq decreases monotonically from 1→0.5
  - Hmarg increases from 0→8
  - Hjoint increases from 0→8
  - Total excitations E decrease from 8→4
- **Correlation:** Across all 256 measurement patterns, the correlation between E and Hmarg is strongly negative (approximately -0.91).
- **Symmetry:** All outcomes with the same r share identical statistics. The multiplicity for each r is C(8,r).

Discussion

Mechanism

The Z-basis effects stem entirely from the Hamming-weight constraint combined with conditioning on the measurement outcome r. The CZ phase gates are invisible to these observables since they only affect relative phases between computational basis states.

Classical Reproducibility

A classical probabilistic mixture uniform over the allowed set reproduces all Z-basis statistics. Entanglement is not certified by Z-only measurements. The quantum nature would only manifest through interference effects.

Phase-Sensitive Extensions

To expose the quantum phases introduced by CZ gates:

1. **Output interference:** Apply Hadamards on qubits 8-15 before measuring in Z

2. **Input interference**: Apply Hadamards on qubits 0-7 before the Z measurement (graph-state approach)
3. Stabilizer correlations: Evaluate multi-qubit Pauli correlators that are sensitive to the CZ-induced phases
   These modifications would break the classical reproducibility and reveal genuine quantum entanglement.

Information-Theoretic Analysis

The multi-information I=Hmarg−Hjoint quantifies the statistical dependence between unmeasured qubits induced by the global constraint. It peaks at intermediate values of r where neither extreme dominates.

Further Considerations

Error Sensitivity

- **Readout bias**: Would shift Eq(r) from the ideal curve, observable as systematic deviation.
- **State preparation errors**: Would modify S(r) and affect Hjoint depending on the error channel.
- **Dephasing**: Leaves Z-basis statistics near ideal but suppresses phase-sensitive correlators.

Applications

- **Constraint-Guided State Preparation**: The protocol can deterministically prepare specific states through post-selection. For example, measuring r=0 guarantees $|11111111\rangle$ on the unmeasured side, though with low yield (2.55×10^{-5}).
- **Quantum Benchmarking**: The excitation-entropy curves provide sensitive benchmarks for device characterization:
  - Deviations from theoretical Eq(r) indicate measurement errors
  - Changes in probability distribution P(r) reveal state preparation fidelity
  - The sharp features at extremes (r=0,8) are particularly diagnostic
- **Entanglement Certification**: While Z-basis measurements alone cannot certify entanglement, adding the interference step transforms this into a genuine entanglement witness protocol.

Conclusions

The constrained 16-qubit system exhibits rich structure in its post-measurement statistics, from a rare but deterministic extreme at r=0 to maximum entropy at r=8, with a smooth excitation-entropy tradeoff between. The Z-basis effects arise purely from combinatorial constraints and measurement-induced conditioning. The CZ entangling layer's role becomes apparent only when interference is introduced to probe relative phases. This work demonstrates how global constraints and local measurements interact to produce nontrivial redistribution of quantum resources, even in bases where entanglement remains hidden.

References

1. Gottesman, D. "The Heisenberg Representation of Quantum Computers." *Proceedings of the XXII International Colloquium on Group Theoretical Methods in Physics*, pp. 32-43 (1998).
2. Nielsen, M.A., Chuang, I.L. *Quantum Computation and Quantum Information*. Cambridge University Press, 10th Anniversary Edition (2010).
3. Hein, M., Eisert, J., Briegel, H.J. "Multiparty entanglement in graph states." *Phys. Rev. A* 69, 062311 (2004).
4. Bärtschi, A., Eidenbenz, S. "Deterministic Preparation of Dicke States." *Proceedings of the 22nd International Conference on Fundamentals of Computation Theory*, pp. 126-139 (2019).
5. Cover, T.M., Thomas, J.A. *Elements of Information Theory*. John Wiley & Sons, 2nd Edition (2006).
6. Preskill, J. "Quantum Computing in the NISQ era and beyond." *Quantum* 2, 79 (2018).